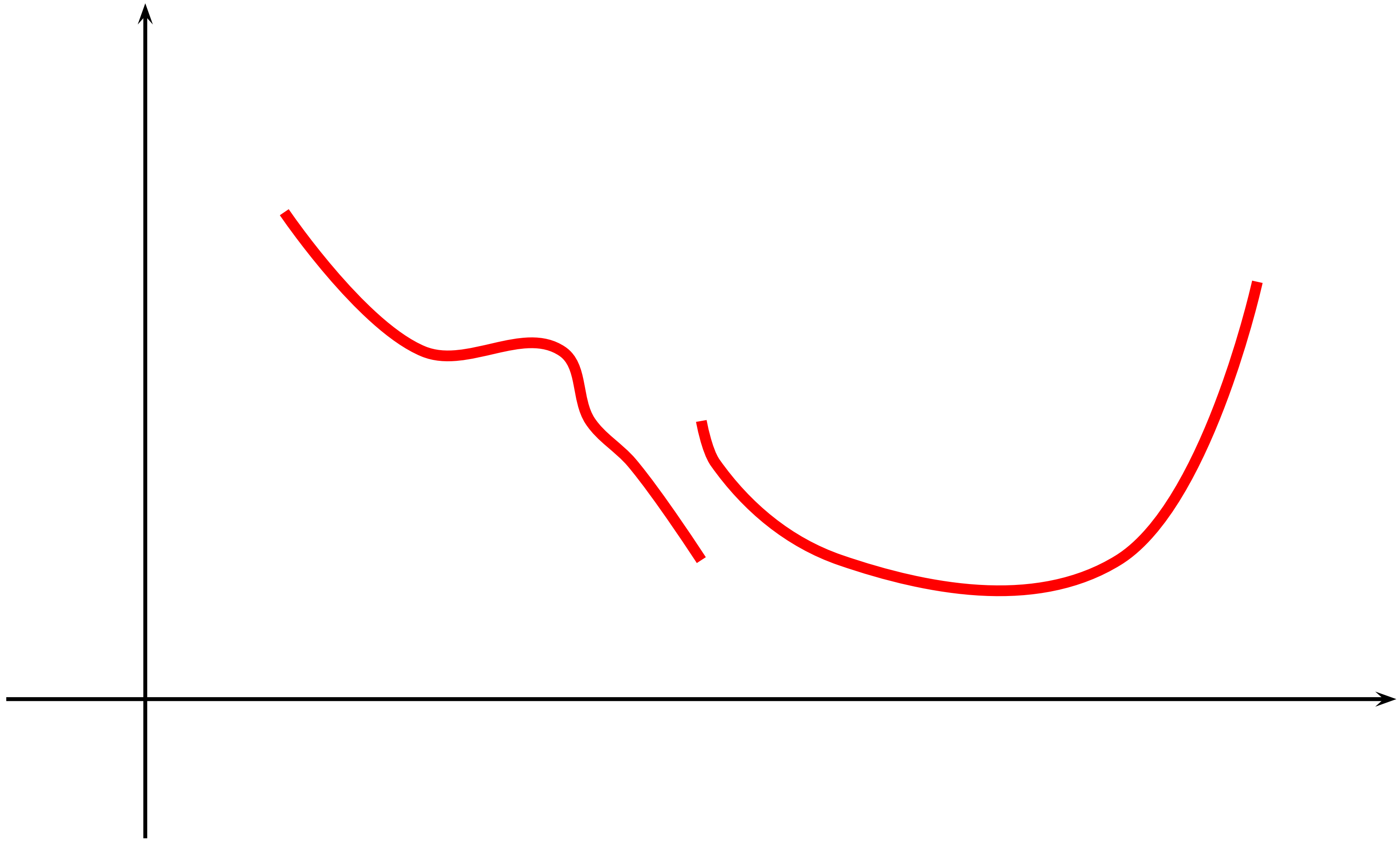
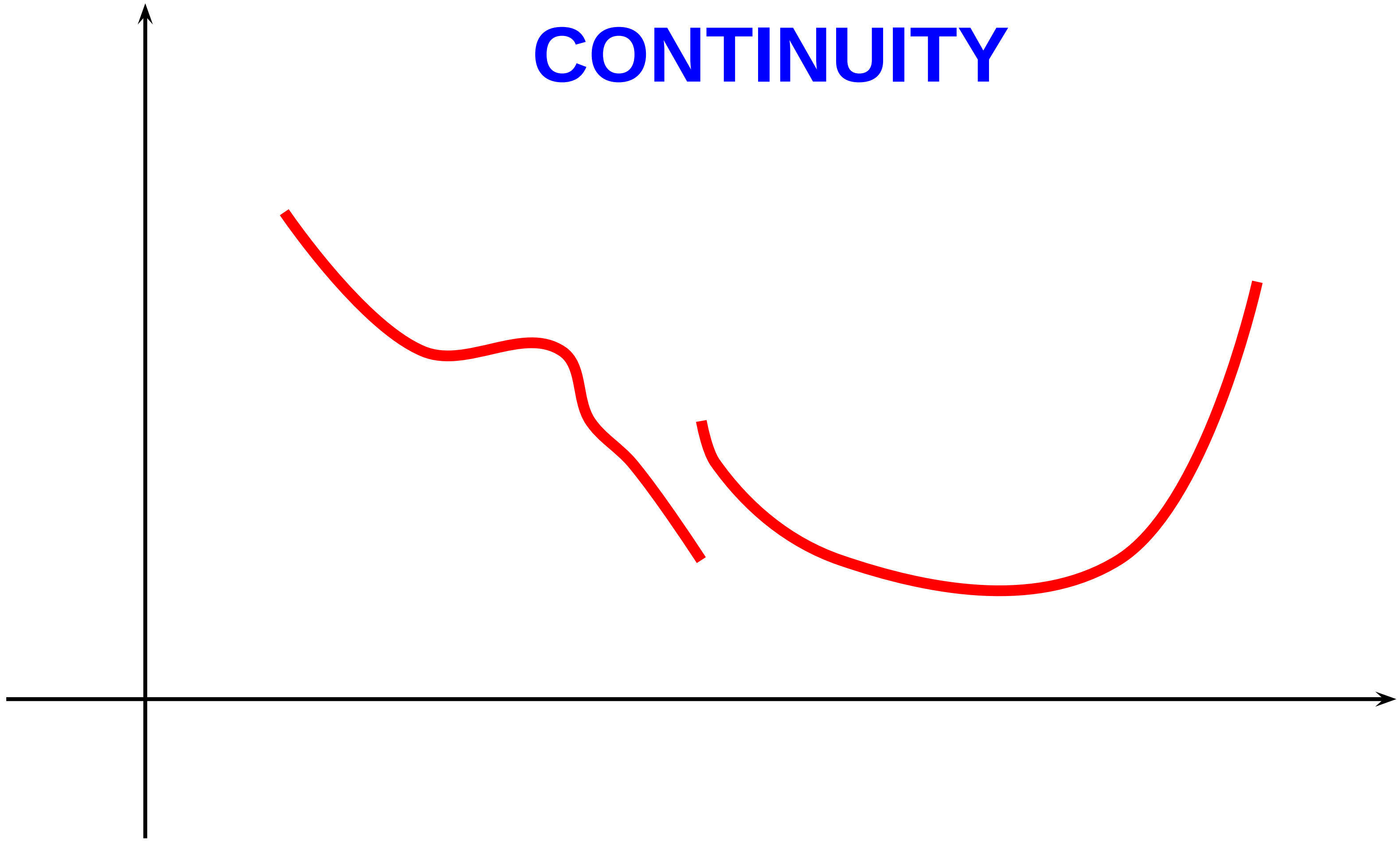
Focus Slide



CONTINUITY



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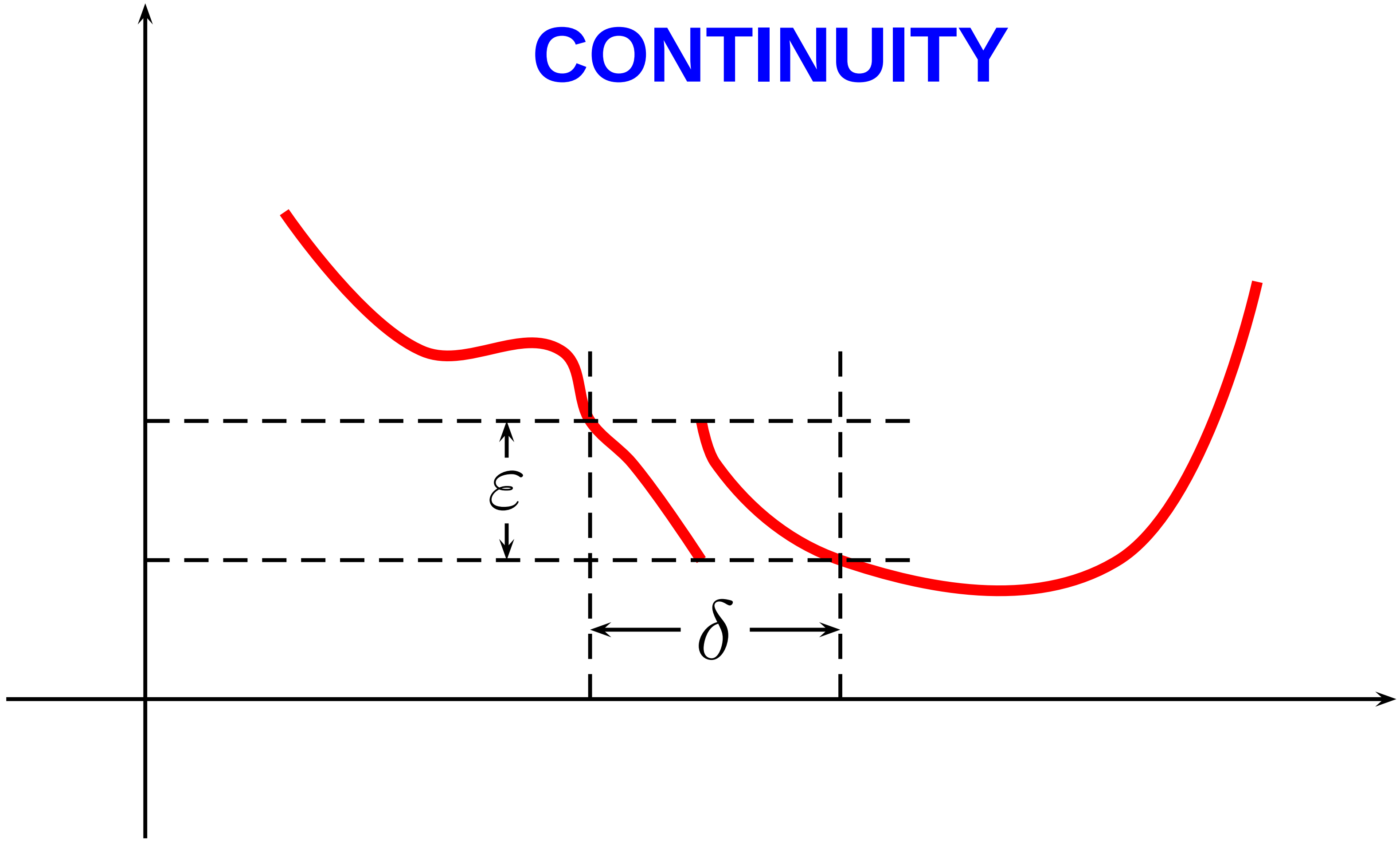
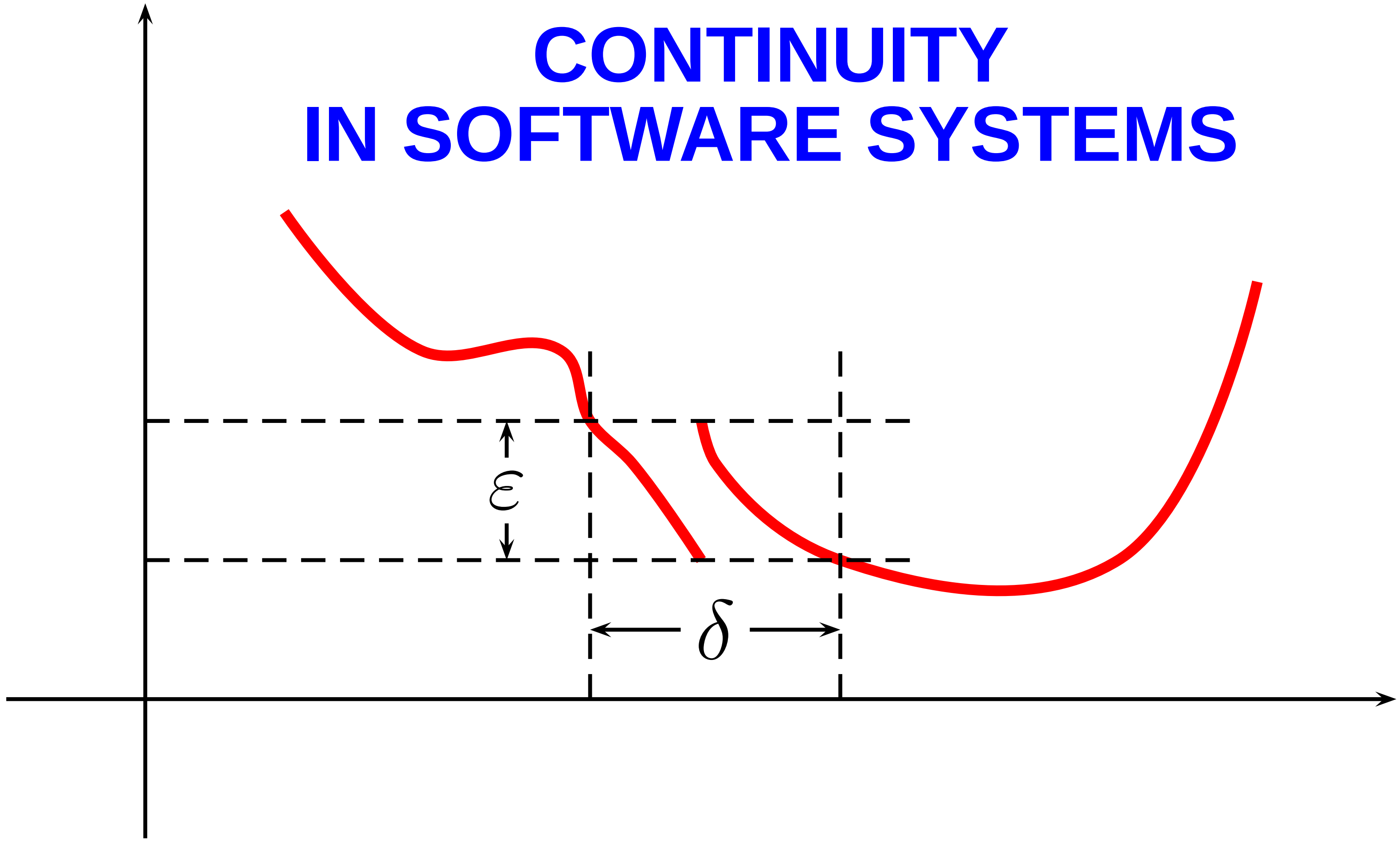


CONTINUITY IN SOFTWARE SYSTEMS



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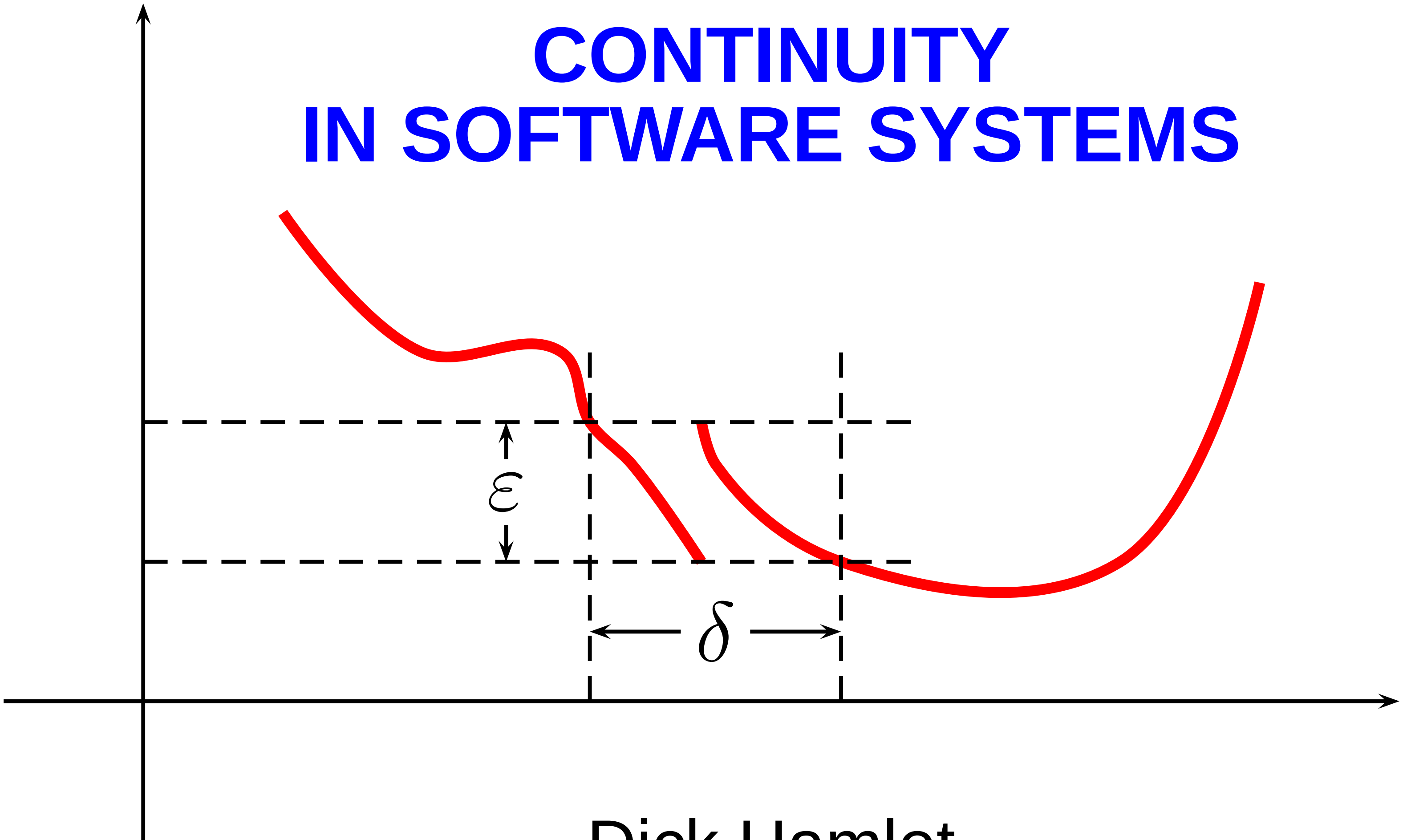

Dick Hamlet
Portland State University
Portland, OR, USA

Outline of the Talk

I. Continuity in the Real World

II. Defining Continuity

III. Testing and Analyzing ‘Continuity’

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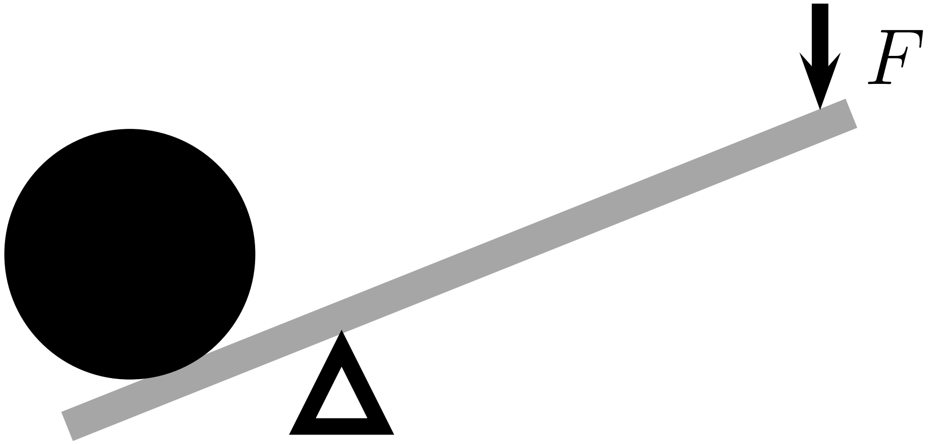

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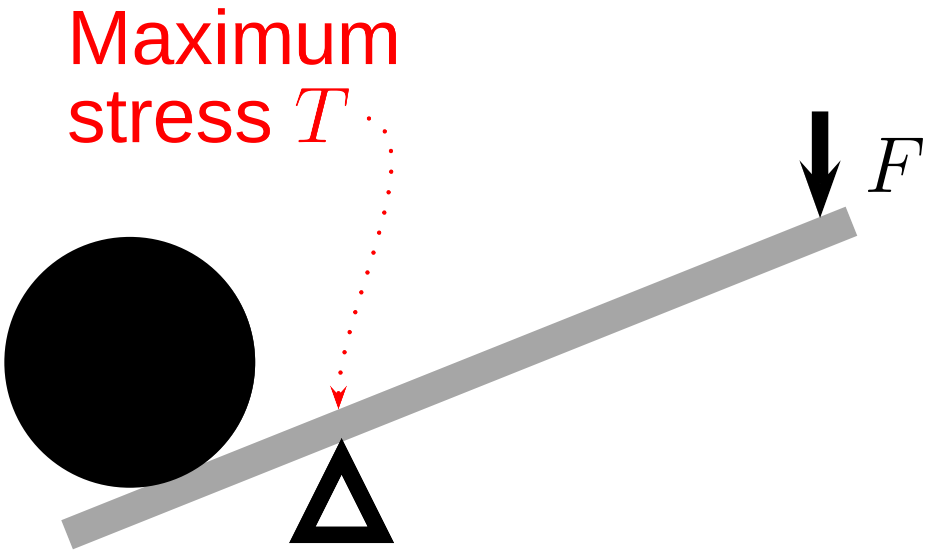

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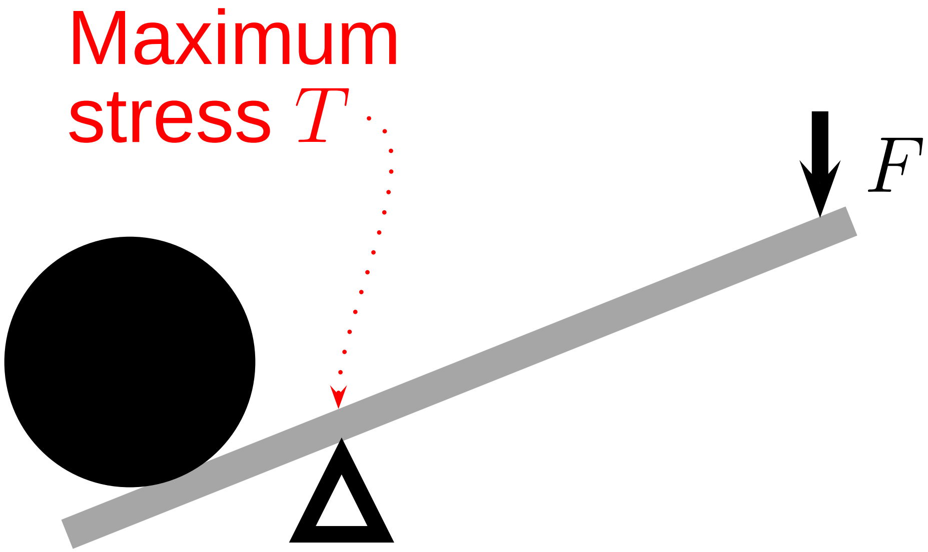
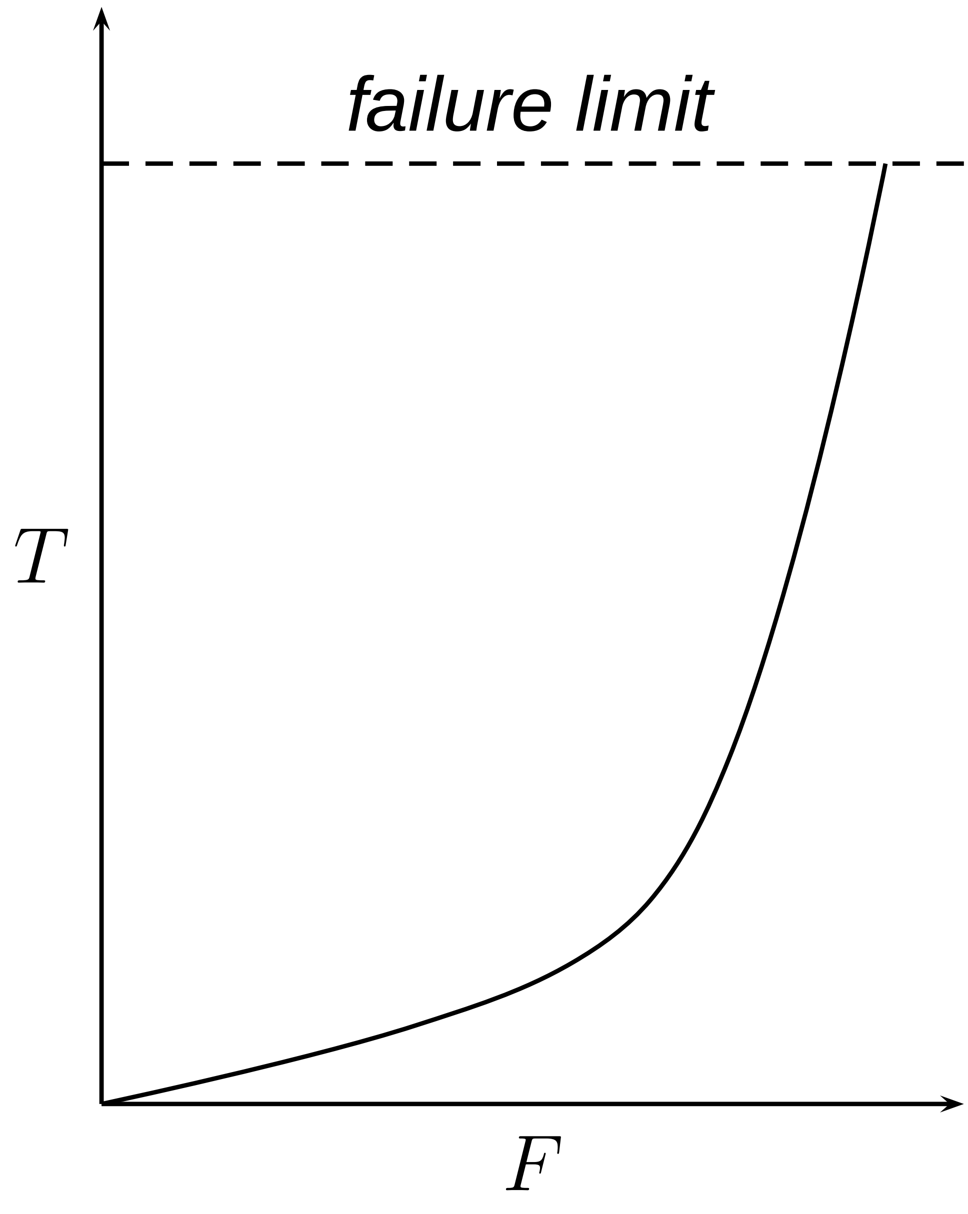
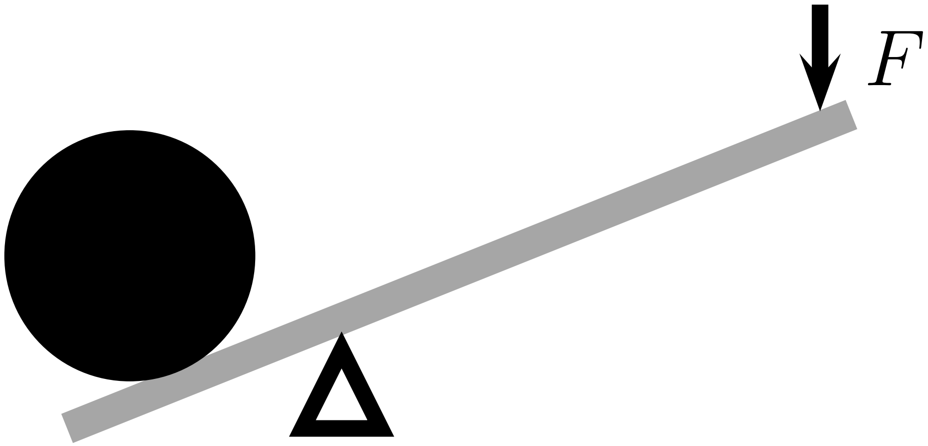




Untrustworthy Behavior



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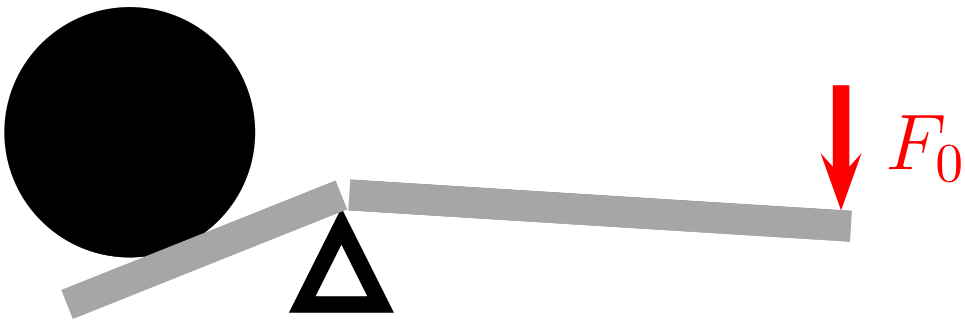

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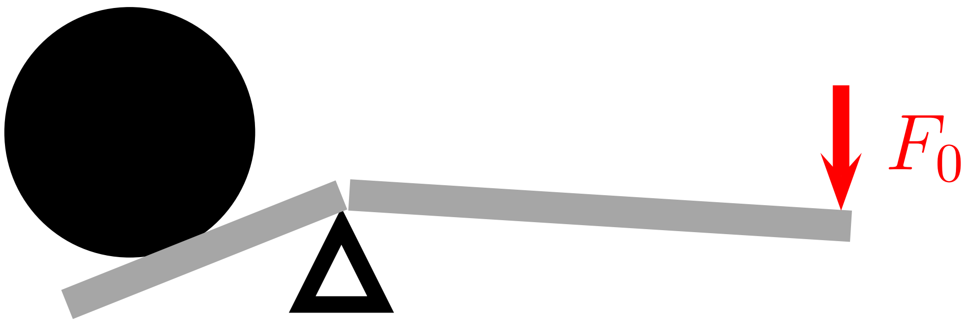
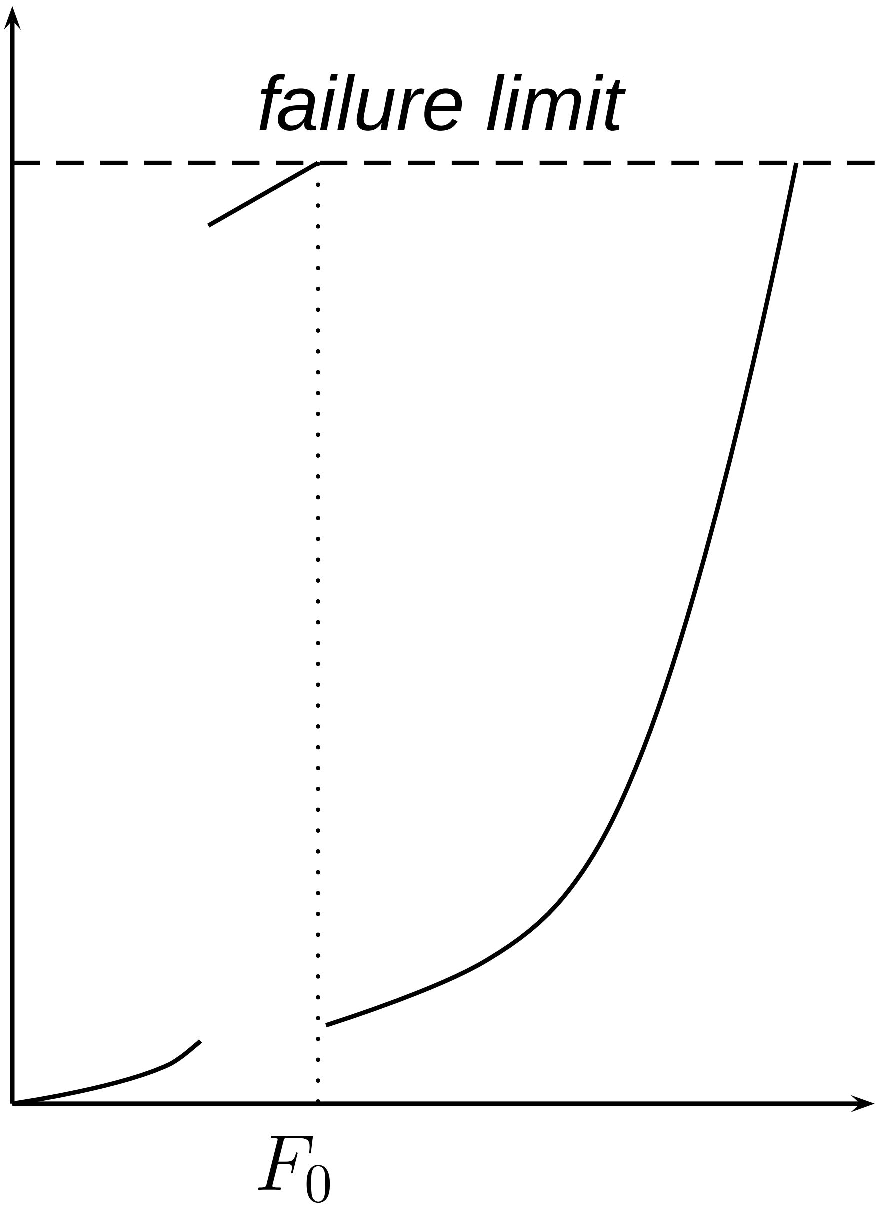
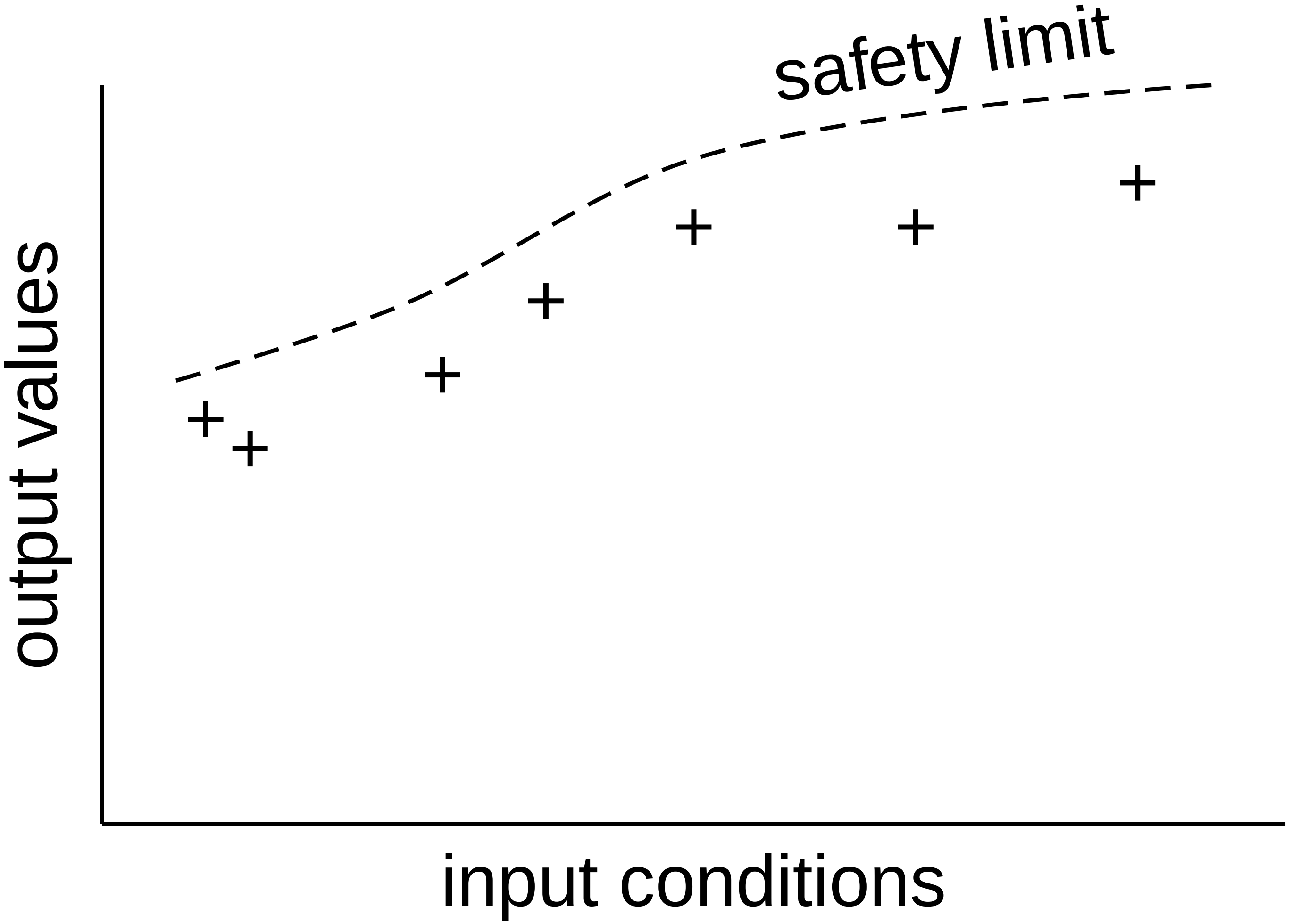




Testing a System for Trustworthiness

Sample the behavior often enough that continuity covers the space between samples



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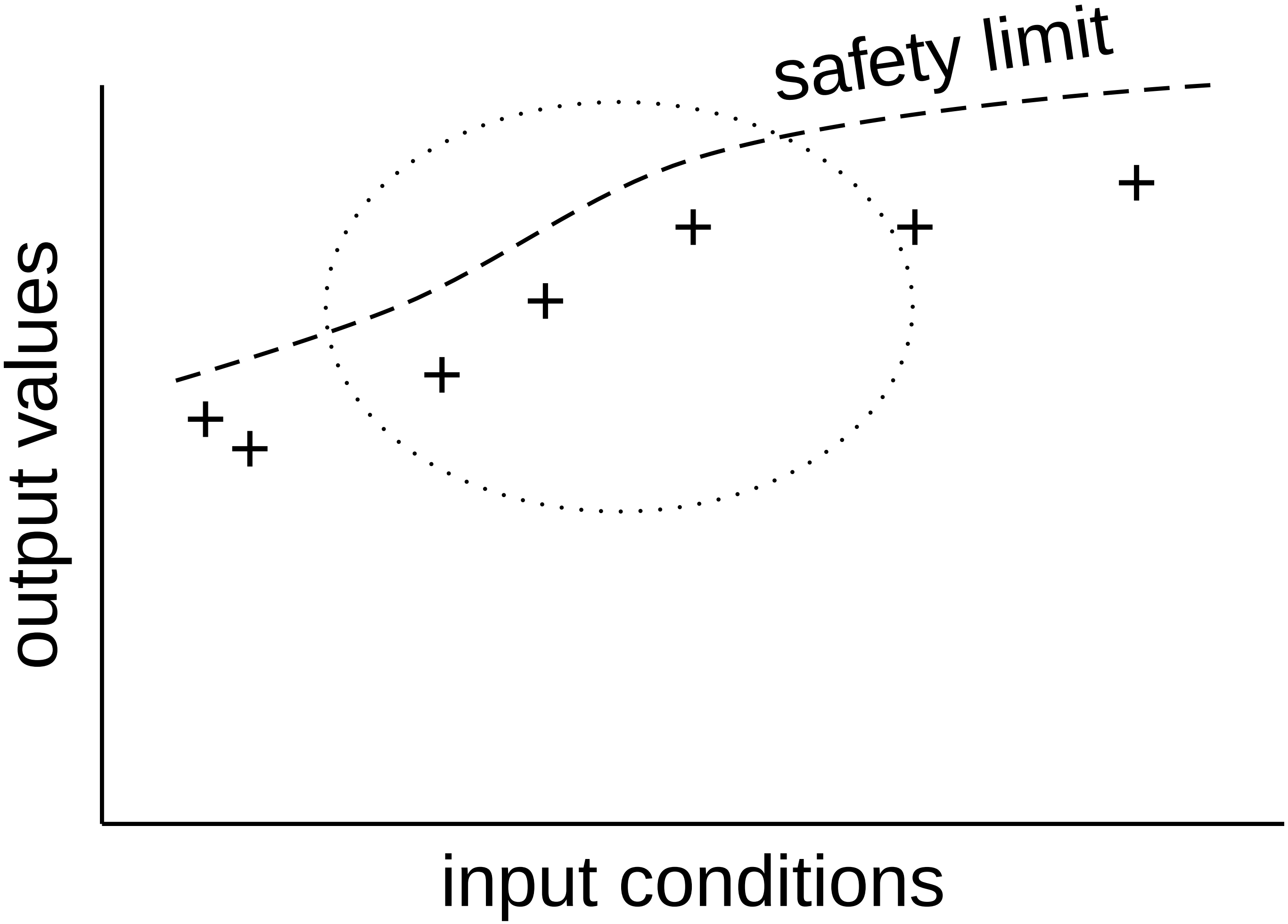
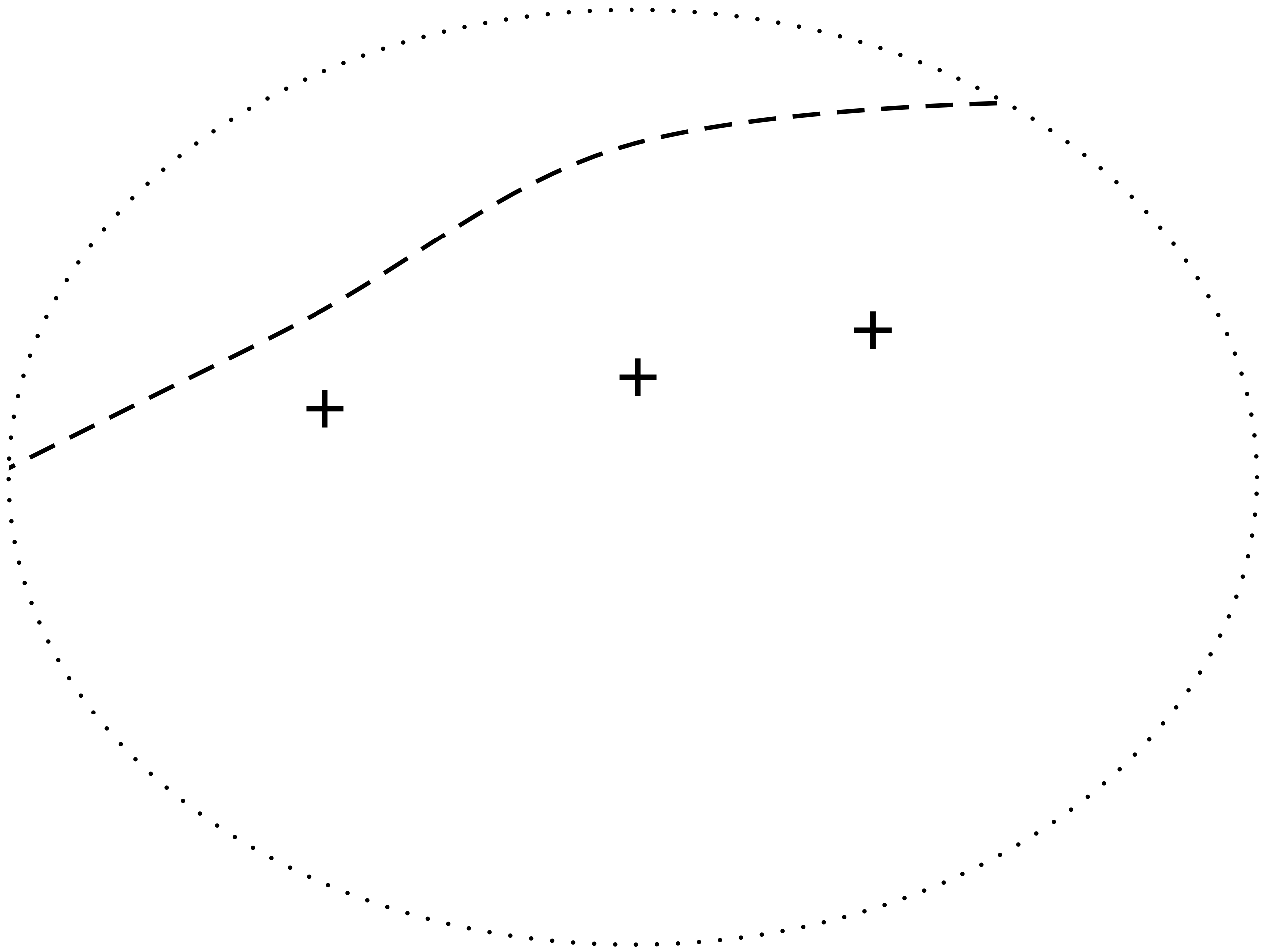


Safety Factors

Continuity isn't enough – something needed like a Lipschitz condition



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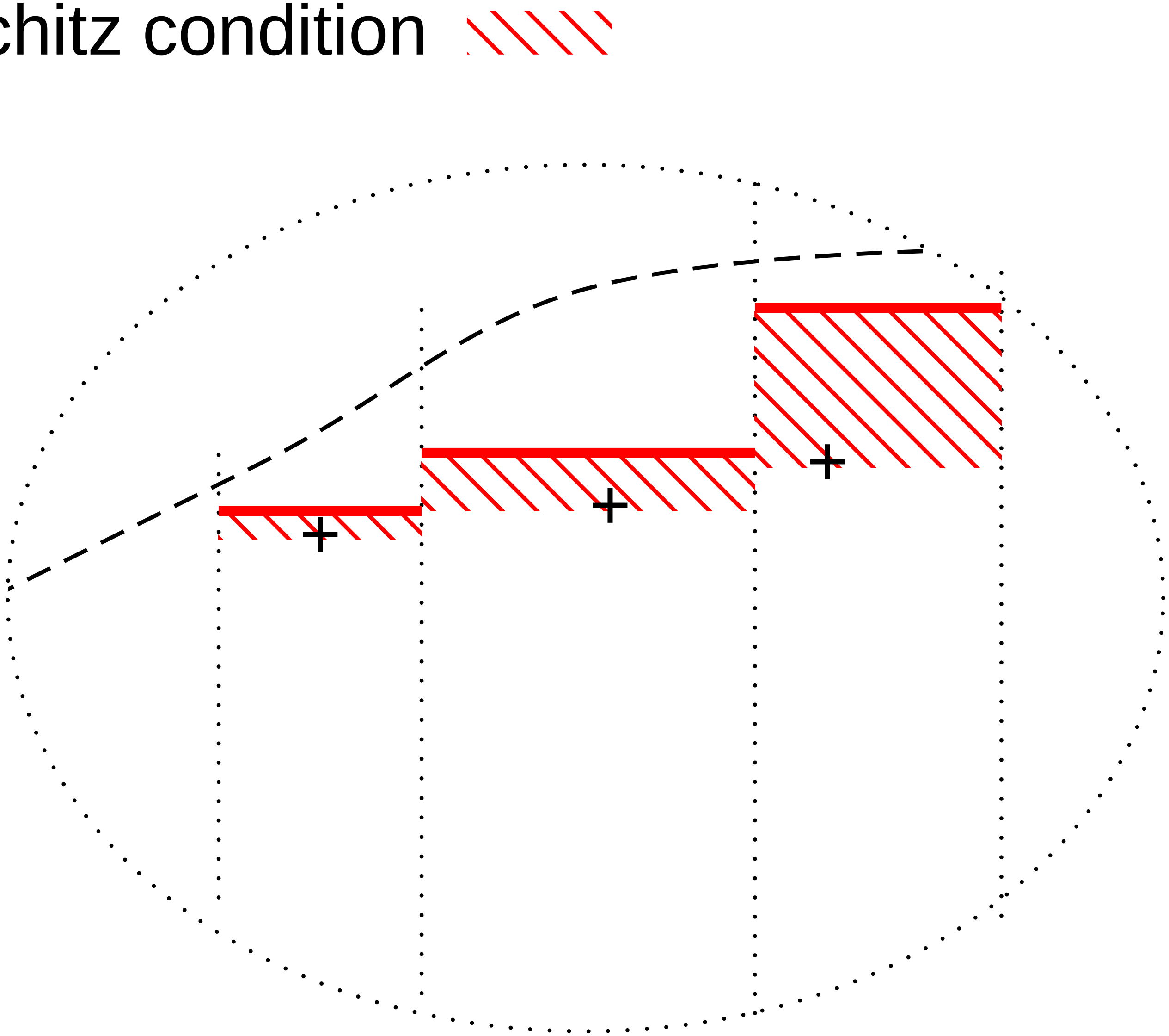

Outline of the Talk

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II. Defining Continuity

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The Real-analysis Definition

The famous '$\varepsilon - \delta$' version:

DEFINITION: A real function f is *continuous* at x_0 iff: Given any $\epsilon > 0,\ \exists\delta > 0$ such that

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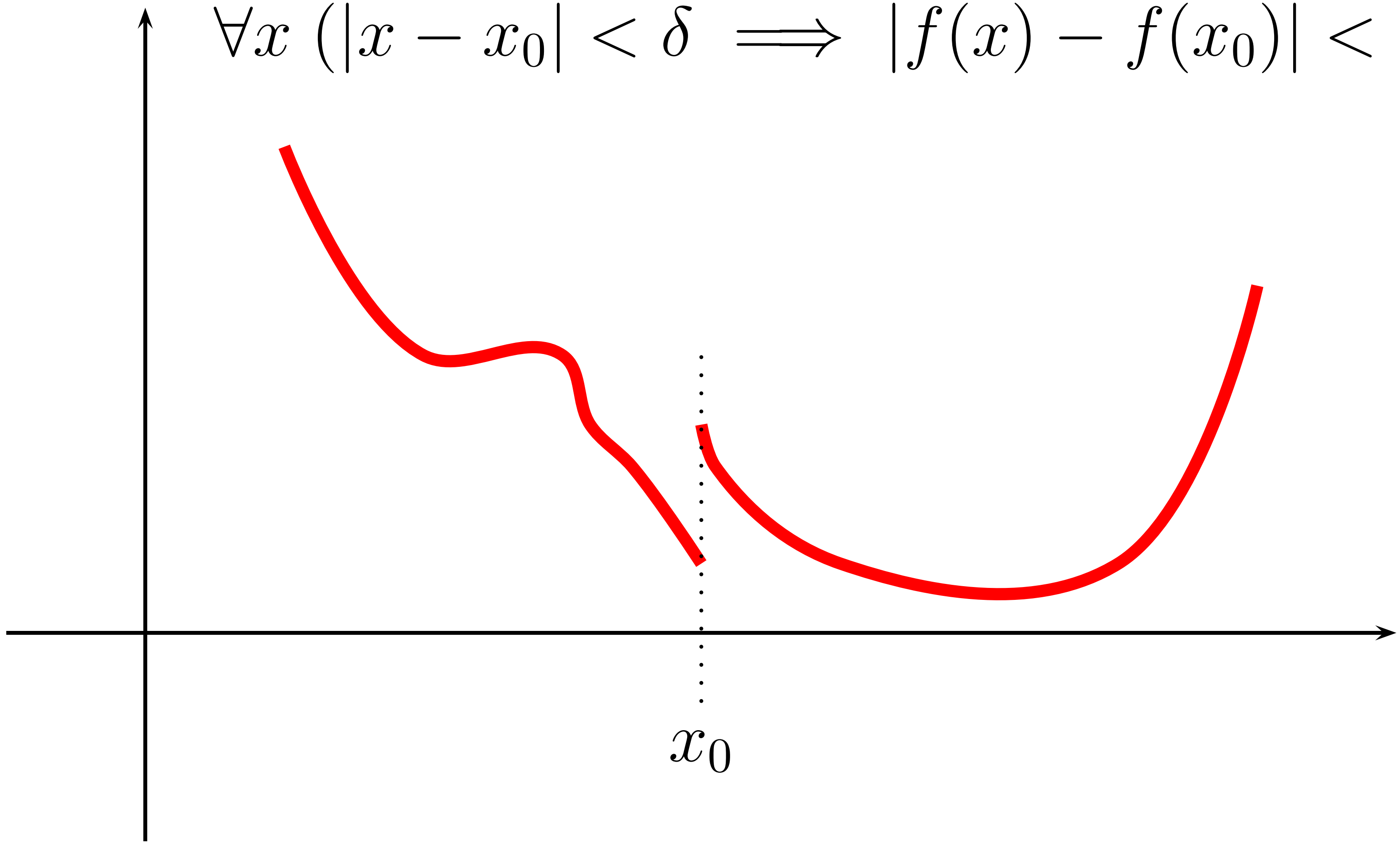

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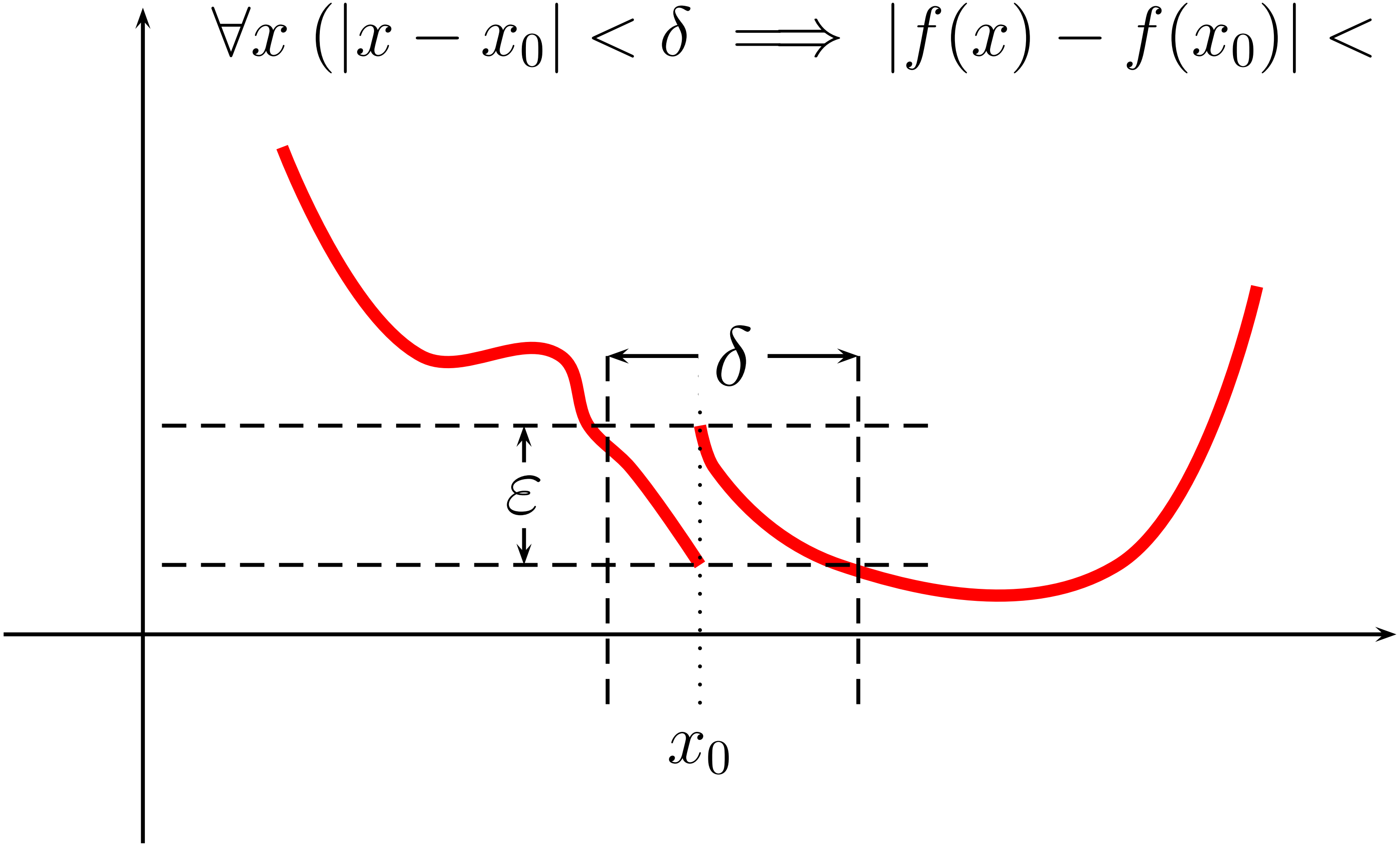

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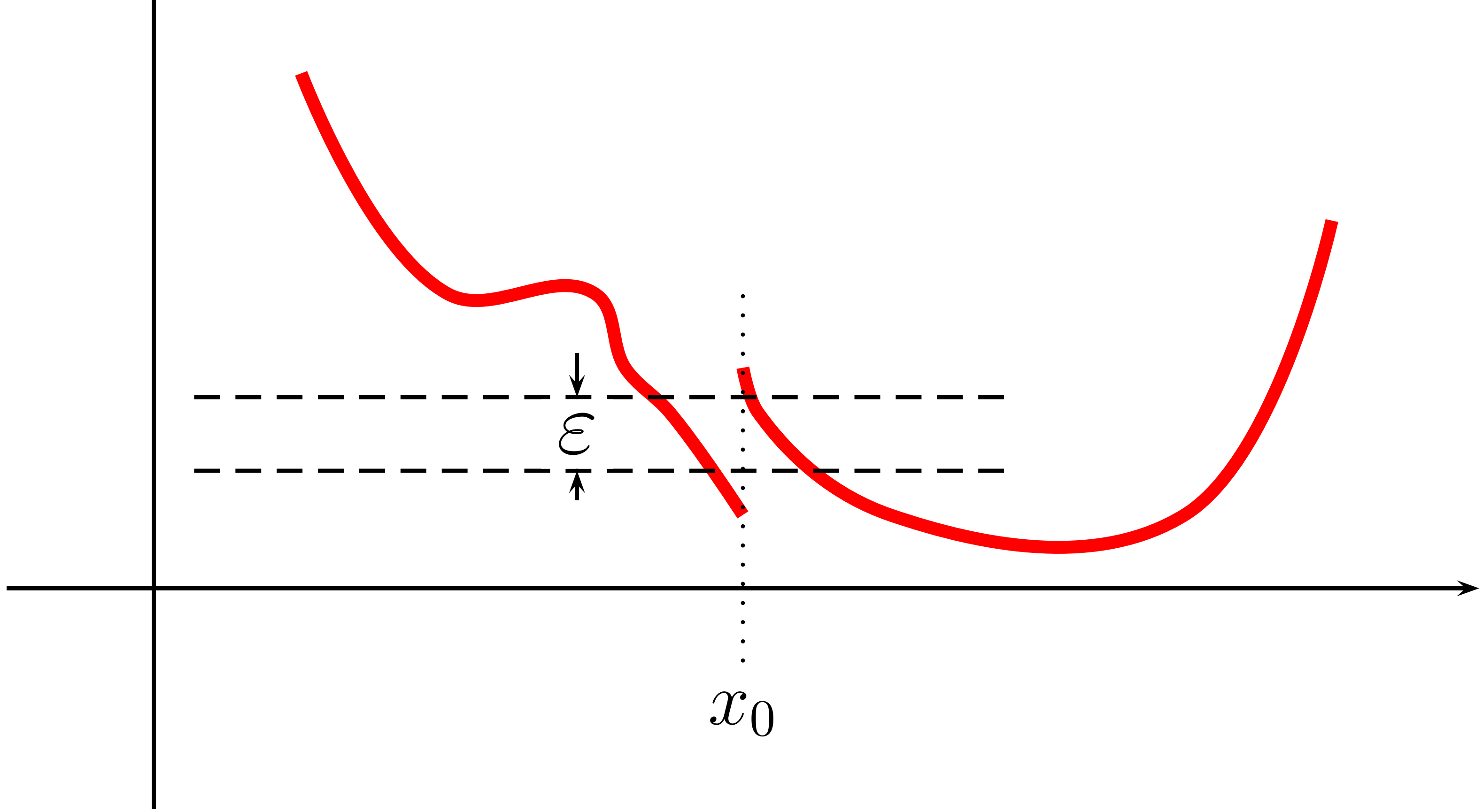
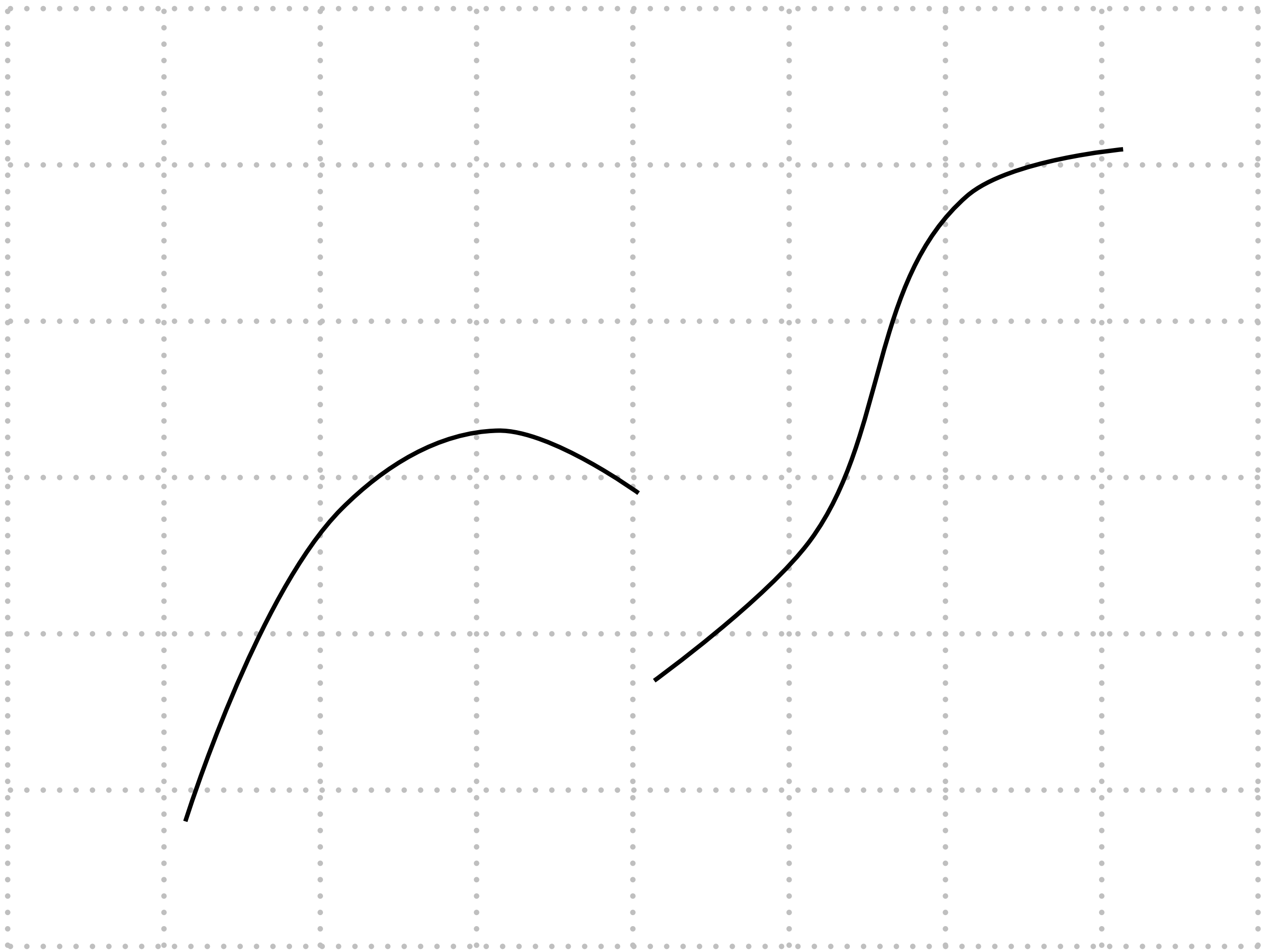


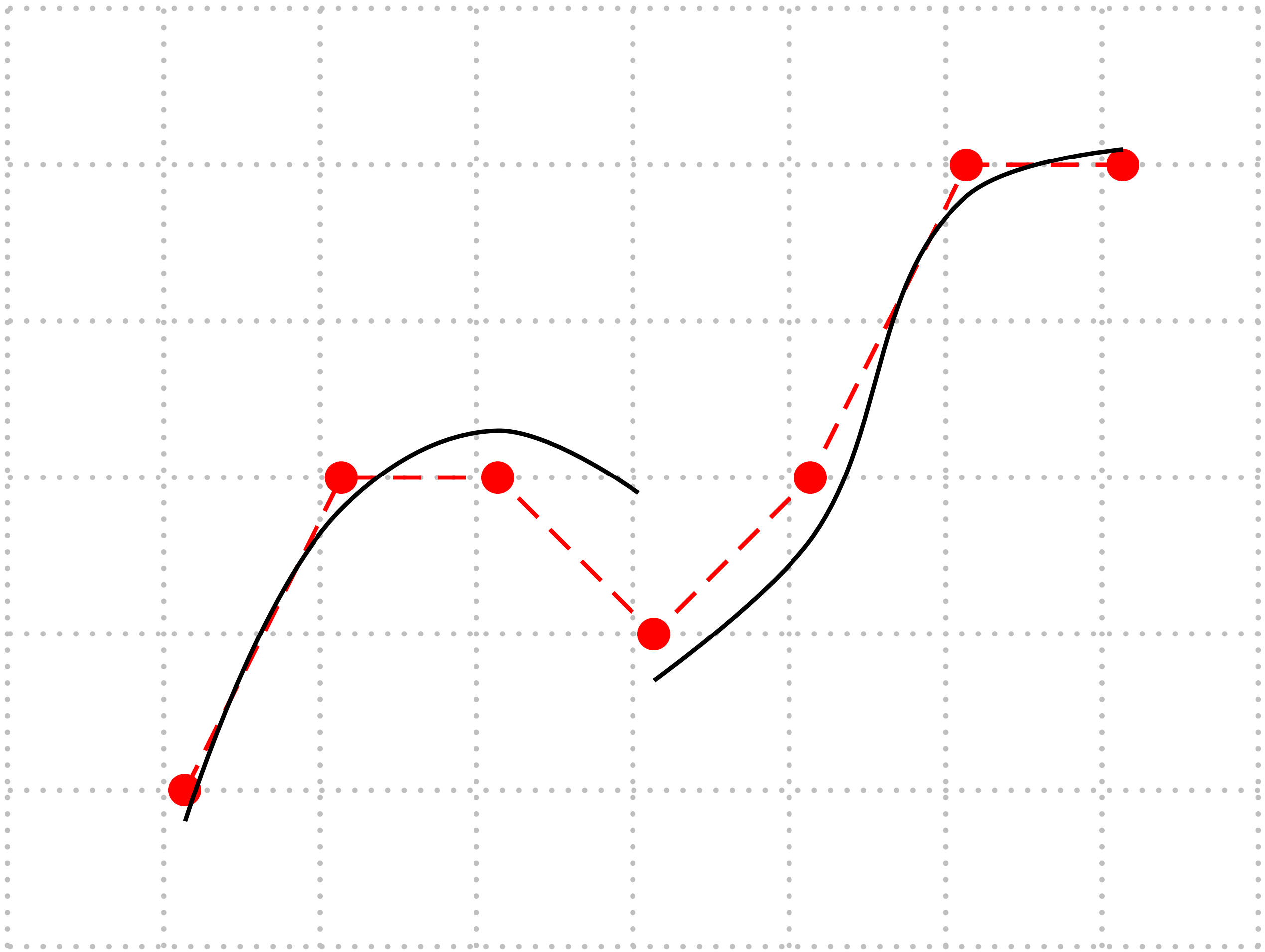
Discrete Functions

Approximating a function $f(\;)$



Discrete Functions

Approximating a function f ( )with a discrete approximation f_d ( ), $f_d(x) = \mathrm{rnd}(f(x))$, integer x



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DEFINITION: An integer function f defined on a finite interval of the integers is *discretely continuous* iff:
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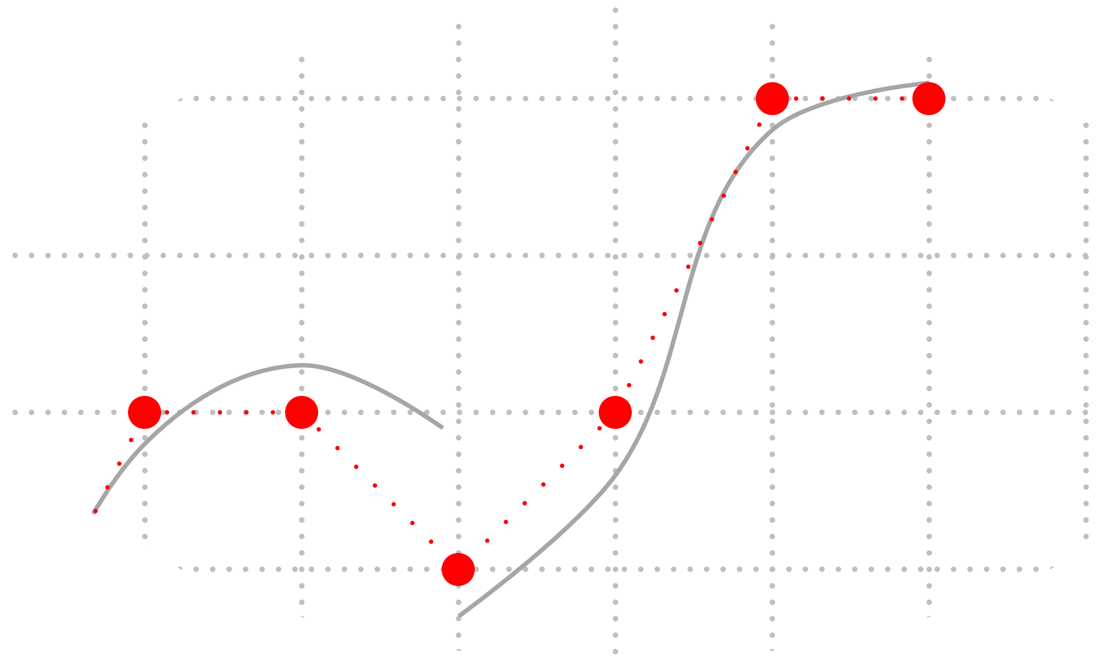

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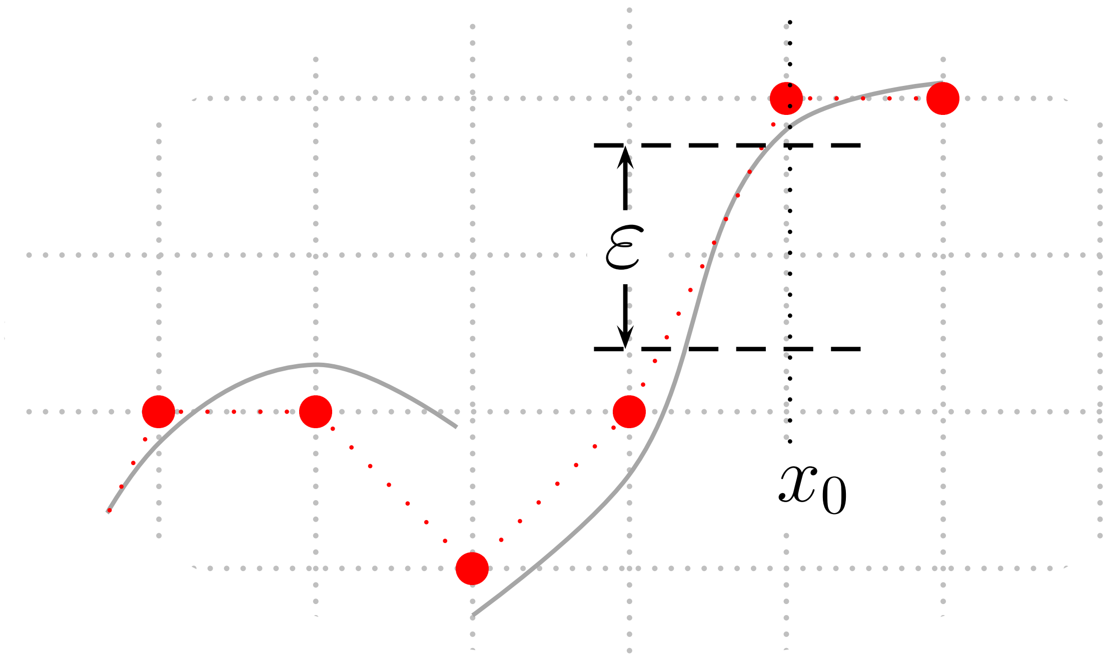

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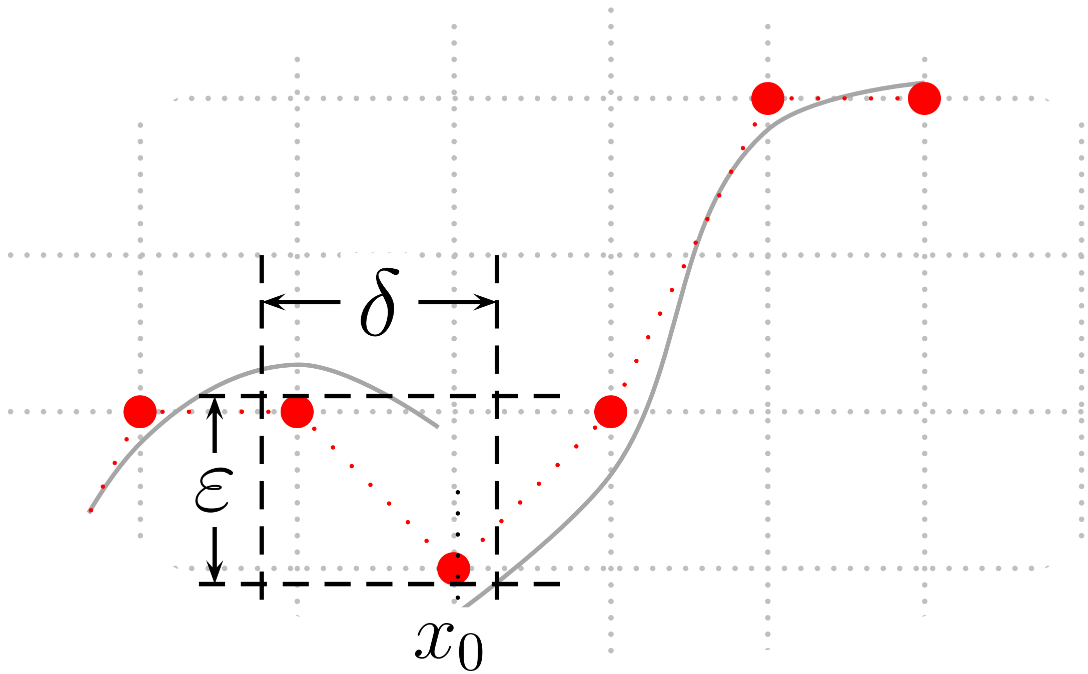

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- are closed under composition
- are *not closed* under arithmetic operations
  - Let $f(x) = x$, for which f_d is discretely continuous everywhere. But $f_d + f_d$ is nowhere discretely continuous.

Floating-point Continuity

A program "computes f to within 1%":

- For all real x, program inputs will approximate x with error at most δ_x, and for all input values t such that $|x - t| < \delta_x$ the program output v_t at t will satisfy $|(f(x) - v_t)/f(x)| < .01$

DEFINITION: The function F computed by a program is floating-point continuous iff it approximates a continuous function to the accuracy of the program's specification.

- Floating-point continuity: almost discrete continuity 'scaled' by floating-point granularity

Failure Continuity

DEFINITION: Program P has specification S. P is *failure continuous* at x_0 iff $\exists b > 0$ such that:

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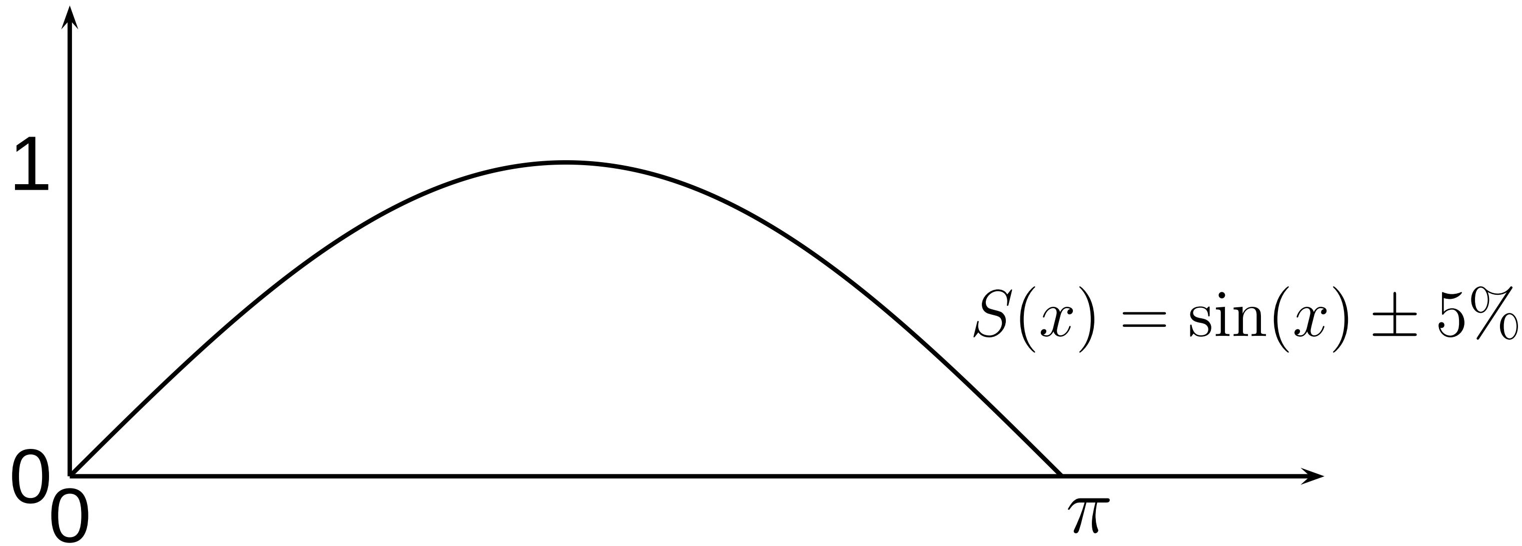

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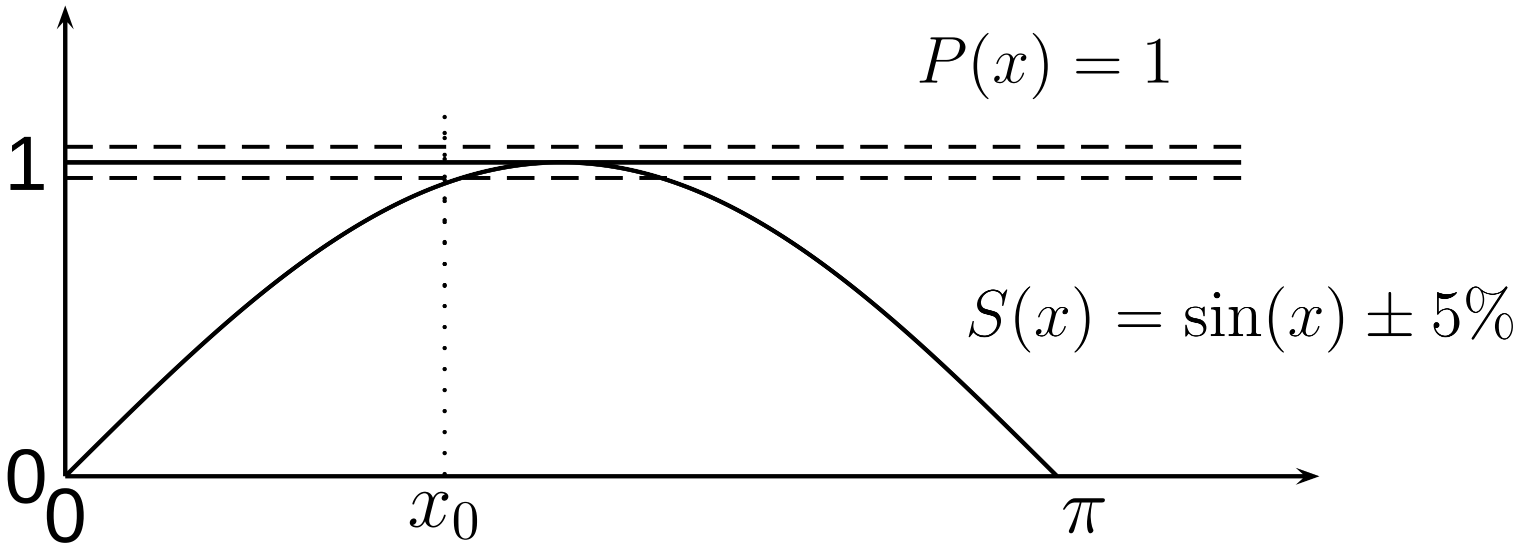

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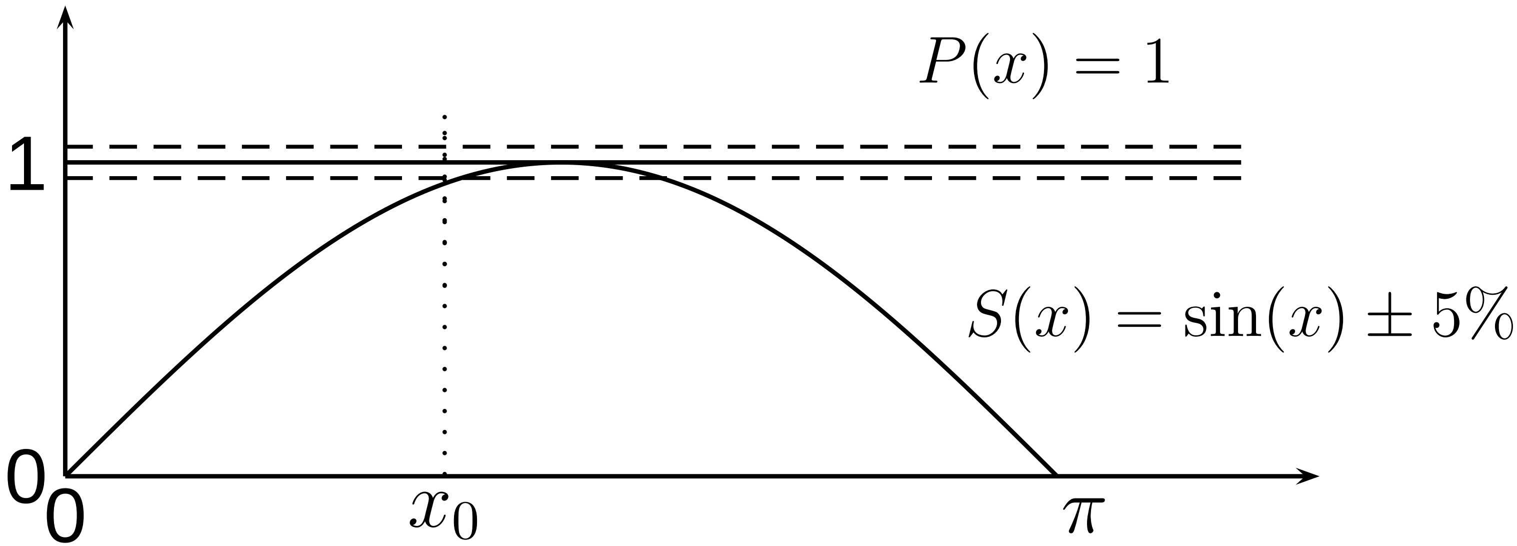

- Failure continuity is what Howden's 'reliable' subdomains have

Program Analysis with Reals Justified

- Program variables are not the real variables we pretend they are

CONJECTURE: If a program computes by symbolic execution a continuous real-valued function, then: (1) The program is discretely continuous over a suitable interval, and (2) There is a specification accuracy for which the program is floating-point continuous.

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- The converse is false

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- Functional languages might be better – program continuities are closed under composition

Ideas to Explore in Continuity Analysis

Suppose a program for a continuous specification *is* continuous.

What new kinds of analysis are possible?

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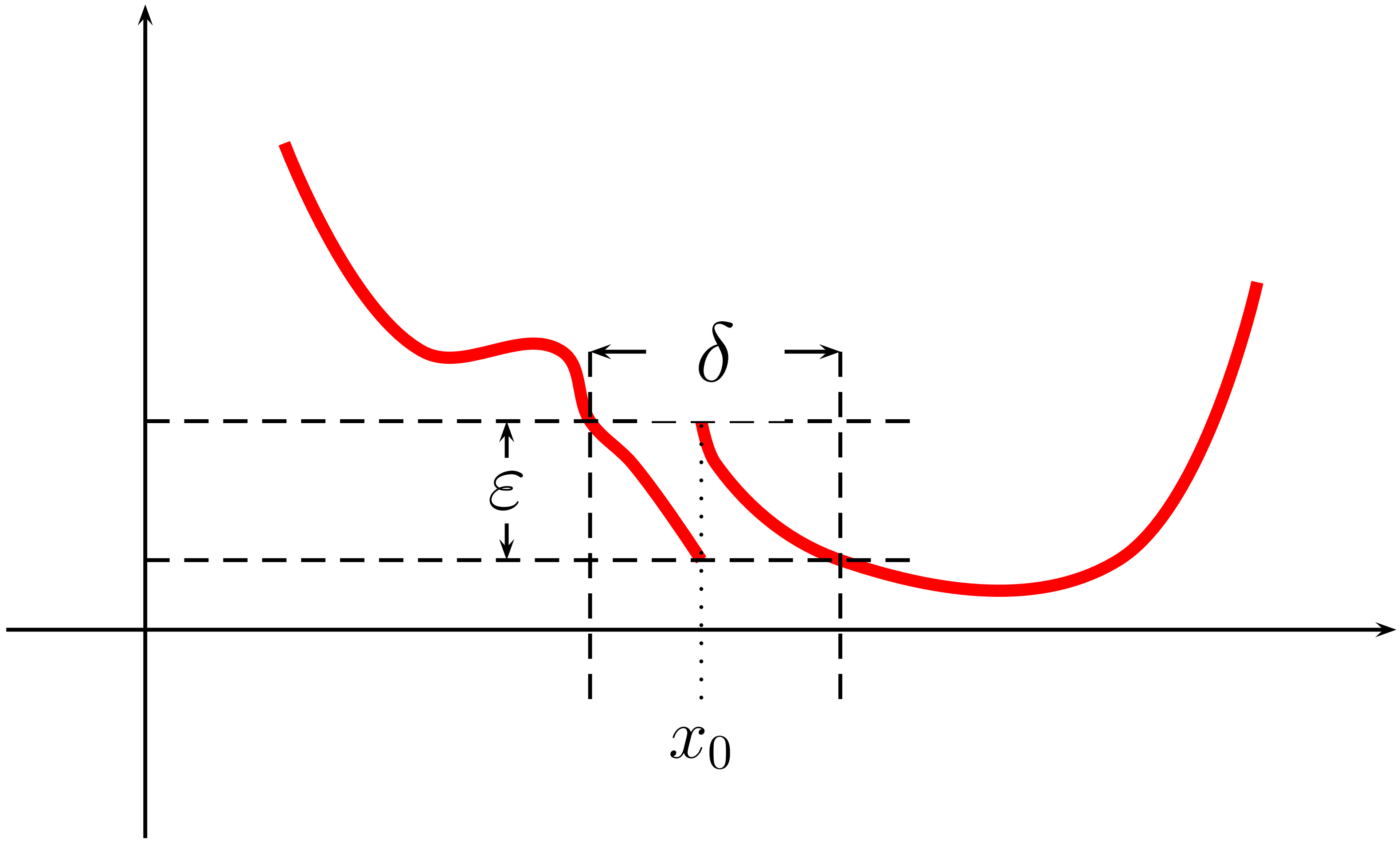
- “Random structural testing” is a name for using a uniform profile on each Lipschitz neighborhood – it may not be intractable in the ultrareliable region
- Exploit continuity in the self-testing/correcting methods of Blum and Ammann

Inherent Discontinuity

- Continuous specifications are important
  - Flight- and process-control software
  - Simulations of natural systems
  - Regulatory-agency problems with software replacing hardware
- But software's forté is *discontinuous* specifications that no other technology can handle
  - Chess-playing robots
  - Compilers and other character-based processors

QUESTIONS?



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